8

Binomial Theorem

TOPIC 1 Binomial Theorem for a Positive Integral Index 'x', Expansion of Binomial, General Term, Coefficient of any Power of 'x'

1. If $\{p\}$ denotes the fractional part of the number p, then $\left\{\frac{3^{200}}{8}\right\}$, is equal to : **[Sep. 06, 2020 (I)]**
   (a) $\frac{5}{8}$ (b) $\frac{7}{8}$
   (c) $\frac{3}{8}$ (d) $\frac{1}{8}$

2. The natural number m, for which the coefficient of x in the binomial expansion of $\left(x^m+\frac{1}{x^2}\right)^{22}$ is 1540, is ______. **[NA Sep. 05, 2020 (I)]**

3. The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^6$ in powers of x, is ____________. **[NA Sep. 05, 2020 (II)]**

4. Let $(2x^2+3x+4)^{10}=\sum_{r=0}^{20} a_r x^r$. Then $\frac{a_7}{a_{13}}$ is equal to ___________. **[NA Sep. 04, 2020 (I)]**

5. If α and β be the coefficients of x^4 and x^2 respectively in the expansion of
   $\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6$, then: **[Jan. 8, 2020 (II)]**
   (a) $\alpha+\beta=60$ (b) $\alpha+\beta=-30$
   (c) $\alpha-\beta=60$ (d) $\alpha-\beta=-132$

6. The smallest natural number n, such that the coefficient of x in the expansion of $\left(x^2+\frac{1}{x^3}\right)^n$ is ${}^nC_{23}$, is : **[April 10, 2019 (II)]**
   (a) 38 (b) 58
   (c) 23 (d) 35

7. If the fourth term in the Binomial expansion of $\left(\frac{2}{x}+x^{\log_8 x}\right)^6$ $(x>0)$ is 20×8^7, then a value of x is: **[April 9, 2019 (I)]**
   (a) 8^3 (b) 8^2
   (c) 8 (d) 8^{-2}

8. If some three consecutive coefficients in the binomial expansion of $(x+1)^n$ in powers of x are in the ratio 2:15:70, then the average of these three coefficients is: **[April 09, 2019 (II)]**
   (a) 964 (b) 232
   (c) 227 (d) 625

9. The sum of the co-efficients of all even degree terms in x in the expansion of $\left(x+\sqrt{x^3-1}\right)^6+\left(x-\sqrt{x^3-1}\right)^6$, $(x>1)$ is equal to : **[April 8, 2019 (I)]**
   (a) 29 (b) 32
   (c) 26 (d) 24

10. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10}x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x>1$, then the value of x is: **[April 08, 2019 (II)]**
    (a) 100 (b) 10
    (c) 10^3 (d) 10^4

11. Let $(x+10)^{50}+(x-10)^{50}=a_0+a_1x+a_2x^2+....+a_{50}x^{50}$, for all $x\in\mathbf{R}$; then $\frac{a_2}{a_0}$ is equal to : **[Jan. 11, 2019 (II)]**
    (a) 12.50 (b) 12.00
    (c) 12.25 (d) 12.75

12. If the third term in the binomial expansion of $\left(1+x^{\log_2 x}\right)^5$ equals 2560, then a possible value of x is: **[Jan. 10, 2019 (I)]**

(a) $\frac{1}{4}$ (b) $4\sqrt{2}$

(c) $\frac{1}{8}$ (d) $2\sqrt{2}$

13. The positive value of λ for which the co-efficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$ is 720, is: **[Jan. 10, 2019 (II)]**

(a) 4 (b) $2\sqrt{2}$

(c) $\sqrt{5}$ (d) 3

14. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to: **[Jan. 9, 2019 (I)]**

(a) 6 (b) 8

(c) 4 (d) 14

15. The coefficient of x^{10} in the expansion of $(1+x)^2(1+x^2)^3(1+x^3)^4$ is equal to **[Online April 15, 2018]**

(a) 52 (b) 44

(c) 50 (d) 56

16. If n is the degree of the polynomial,

$$\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8+\left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$$ and

m is the coefficient of x^n in it, then the ordered pair (n, m) is equal to **[Online April 15, 2018]**

(a) $(12, (20)^4)$ (b) $(8, 5(10)^4)$

(c) $(24, (10)^8)$ (d) $(12, 8(10)^4)$

17. The coefficient of x^2 in the expansion of the product $(2-x^2).((1+2x+3x^2)^6+(1-4x^2)^6)$ is **[Online April 16, 2018]**

(a) 106 (b) 107

(c) 155 (d) 108

18. The sum of the co-efficients of all odd degree terms in the expansion of **[2018]**

$(x+\sqrt{x^3-1})^5+(x-\sqrt{x^3-1})^5, (x>1)$ is :

(a) 0 (b) 1

(c) 2 (d) -1

19. The coefficient of x^{-5} in the binomial expansion of

$$\left(\frac{x+1}{x^{\frac{2}{3}}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right)^{10}$$ where $x\neq 0, 1$, is :

[Online April 9, 2017]

(a) 1 (b) 4

(c) -4 (d) -1

20. If $(27)^{999}$ is divided by 7, then the remainder is : **[Online April 8, 2017]**

(a) 1 (b) 2

(c) 3 (d) 6

21. If the coefficients of x^{-2} and x^{-4} in the expansion of $\left(x^{\frac{1}{3}}+\frac{1}{2x^{\frac{1}{3}}}\right)^{18}$, $(x>0)$, are m and n respectively, then $\frac{m}{n}$ is equal to : **[Online April 10, 2016]**

(a) 27 (b) 182

(c) $\frac{5}{4}$ (d) $\frac{4}{5}$

22. If the coefficients of the three successive terms in the binomial expansion of $(1+x)^n$ are in the ratio 1 : 7 : 42, then the first of these terms in the expansion is: **[Online April 10, 2015]**

(a) 8^{th} (b) 6^{th}

(c) 7^{th} (d) 9^{th}

23. If the coefficents of x^3 and x^4 in the expansion of $(1+ax+bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to: **[2014]**

(a) $\left(14,\frac{272}{3}\right)$ (b) $\left(16,\frac{272}{3}\right)$

(c) $\left(16,\frac{251}{3}\right)$ (d) $\left(14,\frac{251}{3}\right)$

24. If $X=\{4^n-3n-1 : n\in N\}$ and $Y=\{9(n-1): n\in N\}$, where N is the set of natural numbers, then $X\cup Y$ is equal to: **[2014]**

(a) X (b) Y

(c) N (d) $Y-X$

25. If $1+x^4+x^5=\sum_{i=0}^{5}a_i(1+x)^i$, for all x in R, then a_2 is: **[Online April 12, 2014]**

(a) -4 (b) 6

(c) -8 (d) 10

26. If $\left(2+\frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal, then these terms are: **[Online April 12, 2014]**

(a) 7^{th} and 8^{th} (b) 8^{th} and 9^{th}

(c) 28^{th} and 29^{th} (d) 27^{th} and 28^{th}

27. The number of terms in the expansion of $(1+x)^{101}(1+x^2-x)^{100}$ in powers of x is: **[Online April 9, 2014]**

(a) 302 (b) 301

(c) 202 (d) 101

28. If for positive integers $r > 1$, $n > 2$, the coefficients of the $(3r)^{th}$ and $(r+2)^{th}$ powers of x in the expansion of $(1+x)^{2n}$ are equal, then n is equal to : **[Online April 25, 2013]**
(a) $2r+1$ (b) $2r-1$
(c) $3r$ (d) $r+1$

29. The sum of the rational terms in the binomial expansion of $\left(2^{\frac{1}{2}}+3^{\frac{1}{5}}\right)^{10}$ is : **[Online April 23, 2013]**
(a) 25 (b) 32
(c) 9 (d) 41

30. If the 7th term in the binomial expansion of $\left(\frac{3}{\sqrt[3]{84}}+\sqrt{3}\ln x\right)^{9}$, $x > 0$, is equal to 729, then x can be : **[Online April 22, 2013]**
(a) e^2 (b) e
(c) $\frac{e}{2}$ (d) $2e$

31. If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is : **[2012]**
(a) an irrational number
(b) an odd positive integer
(c) an even positive integer
(d) a rational number other than positive integers

32. The number of terms in the expansion of $\left(y^{1/5}+x^{1/10}\right)^{55}$, in which powers of x and y are free from radical signs are **[Online May 12, 2012]**
(a) six (b) twelve
(c) seven (d) five

33. If $f(y) = 1-(y-1)+(y-1)^2-(y-1)^3 + ... - (y-1)^{17}$, then the coefficient of y^2 in it is **[Online May 7, 2012]**
(a) ${}^{17}C_2$ (b) ${}^{17}C_3$
(c) ${}^{18}C_2$ (d) ${}^{18}C_3$

34. **Statement - 1 :** For each natural number n, $(n + 1)^7-1$ is divisible by 7.

Statement - 2 : For each natural number $n, n^7 - n$ is divisible by 7. **[2011 RS]**
(a) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(b) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is true, Statement-2 is false
(d) Statement-1 is false, Statement-2 is true

35. The coefficient of x^7 in the expansion of $(1-x-x^2+x^3)^6$ is **[2011]**
(a) –132 (b) –144
(c) 132 (d) 144

36. The remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9 is: **[2009]**
(a) 2 (b) 7
(c) 8 (d) 0

37. **Statement -1 :** $\sum_{r=0}^{n}(r+1)\ {}^{n}C_r = (n+2)2^{n-1}$.

Statement-2: $\sum_{r=0}^{n}(r+1)\ {}^{n}C_r x^r = (1+x)^n + nx(1+x)^{n-1}$. **[2008]**
(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

38. In the binomial expansion of $(a-b)^n$, $n \geq 5$, the sum of 5^{th} and 6^{th} terms is zero, then a/b equals **[2007]**
(a) $\frac{n-5}{6}$ (b) $\frac{n-4}{5}$
(c) $\frac{5}{n-4}$ (d) $\frac{6}{n-5}$.

39. For natural numbers m, n if $(1-y)^m(1+y)^n = 1+a_1y+a_2y^2+.......$ and $a_1 = a_2 = 10$, then (m,n) is **[2006]**
(a) $(20,45)$ (b) $(35,20)$
(c) $(45,35)$ (d) $(35,45)$

40. If the coefficient of x^7 in $\left[ax^2+\left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax-\left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation **[2005]**
(a) $a-b=1$ (b) $a+b=1$
(c) $\frac{a}{b}=1$ (d) $ab=1$

41. The coefficient of x^n in expansion of $(1+x)(1-x)^n$ is **[2004]**
(a) $(-1)^{n-1}n$ (b) $(-1)^n(1-n)$
(c) $(-1)^{n-1}(n-1)^2$ (d) $(n-1)$

42. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is **[2003]**

(a) 35 (b) 32
(c) 33 (d) 34

43. r and n are positive integers $r > 1$, $n > 2$ and coefficient of $(r+2)^{\text{th}}$ term and $3r^{\text{th}}$ term in the expansion of $(1+x)^{2n}$ are equal, then n equals **[2002]**

(a) $3r$ (b) $3r+1$
(c) $2r$ (d) $2r+1$

44. The coefficients of x^p and x^q in the expansion of $(1+x)^{p+q}$ are **[2002]**

(a) equal
(b) equal with opposite signs
(c) reciprocals of each other
(d) none of these

TOPIC 2 Middle Term, Greatest Term, Independent Term, Particular Term from end in Binomial Expansion, Greatest Binomial Coefficients

45. If the constant term in the binomial expansion of $\left(\sqrt{x}\,\frac{k}{x^2}\right)^{10}$ is 405, then $|k|$ equals: **[Sep. 06, 2020 (II)]**

(a) 9 (b) 1
(c) 3 (d) 2

46. If for some positive integer n, the coefficients of three consecutive terms in the binomial expansion of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14, then the largest coefficient in this expansion is : **[Sep. 04, 2020 (II)]**

(a) 462 (b) 330
(c) 792 (d) 252

47. If the number of integral terms in the expansion of $(3^{1/2}+5^{1/8})^n$ is exactly 33, then the least value of n is : **[Sep. 03, 2020 (I)]**

(a) 264 (b) 128
(c) 256 (d) 248

48. If the term independent of x in the expansion of $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is k, then $18k$ is equal to : **[Sep. 03, 2020 (II)]**

(a) 5 (b) 9
(c) 7 (d) 11

49. Let $\alpha > 0, \beta > 0$ be such that $\alpha^3+\beta^2=4$. If the maximum value of the term independent of x in the binomial expansion of $(\alpha x^{\frac{1}{9}}+\beta x^{-\frac{1}{6}})^{10}$ is $10k$, then k is equal to : **[Sep. 02, 2020 (I)]**

(a) 336 (b) 352
(c) 84 (d) 176

50. For a positive integer n, $\left(1+\frac{1}{x}\right)^n$ is expanded in increasing powers of x. If three consecutive coefficients in this expansion are in the ratio, 2 : 5 : 12, then n is equal to __________. **[NA Sep. 02, 2020 (II)]**

51. In the expansion of $\left(\frac{x}{\cos\theta}+\frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8}\le\theta\le\frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16}\le\theta\le\frac{\pi}{8}$, then the ratio $l_2 : l_1$ is equal to : **[Jan. 9, 2020 (II)]**

(a) $1:8$ (b) $16:1$
(c) $8:1$ (d) $1:16$

52. The total number is irrational terms in the binomial expansion of $\left(7^{\frac{1}{5}}-3^{\frac{1}{10}}\right)^{60}$ is : **[Jan. 12, 2019 (II)]**

(a) 55 (b) 49
(c) 48 (d) 54

53. A ratio of the 5th term from the begining to the 5th term from the end in the binomial expansion of $\left(2^{\frac{1}{3}}+\frac{1}{2(3)^{\frac{1}{3}}}\right)^{10}$ is: **[Jan. 12, 2019 (I)]**

(a) $1:2(6)^{\frac{1}{3}}$ (b) $1:4(16)^{\frac{1}{3}}$
(c) $4(36)^{\frac{1}{3}}:1$ (d) $2(36)^{\frac{1}{3}}:1$

54. The term independent of x in the binomial expansion of $\left(1-\frac{1}{x}+3x^5\right)\left(2x^2-\frac{1}{x}\right)^8$ is : **[Online April 11, 2015]**

(a) 496 (b) –496
(c) 400 (d) –400

55. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is **[2013]**

(a) 4 (b) 120
(c) 210 (d) 310

56. The ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(x^2+\frac{2}{x}\right)^{15}$ is : **[Online April 9, 2013]**

(a) 7 : 16 (b) 7 : 64
(c) 1 : 4 (d) 1 : 32

57. The middle term in the expansion of $\left(1-\frac{1}{x}\right)^n(1-x)^n$ in powers of x is **[Online May 26, 2012]**

(a) $-{}^{2n}C_{n-1}$ (b) $-{}^{2n}C_n$
(c) ${}^{2n}C_{n-1}$ (d) ${}^{2n}C_n$

58. The coefficient of the middle term in the binomial expansion in powers of x of $(1+\alpha x)^4$ and of $(1-\alpha x)^6$ is the same if α equals **[2004]**

(a) $\frac{3}{5}$ (b) $\frac{10}{3}$
(c) $\frac{-3}{10}$ (d) $\frac{-5}{3}$

TOPIC 3 Properties of Binomial Coefficients, Number of Terms in the Expansion of $(x+y+z)^n$, Binomial theorem for any Index, Multinomial theorem, Infinite Series

59. The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to : **[Sep. 04, 2020 (I)]**

(a) ${}^{51}C_7 - {}^{30}C_7$ (b) ${}^{50}C_7 - {}^{30}C_7$
(c) ${}^{50}C_6 - {}^{30}C_6$ (d) ${}^{51}C_7 + {}^{30}C_7$

60. The coefficient of x^4 in the expansion of $(1+x+x^2)^{10}$ is ________. **[NA Jan. 9, 2020 (I)]**

61. If the sum of the coefficients of all even powers of x in the product $(1+x+x^2+...+x^{2n})(1-x+x^2-x^3+...+x^{2n})$ is 61, then n is equal to ———. **[NA Jan. 7, 2020 (I)]**

62. The term independent of x in the expansion of $\left(\frac{1}{60}-\frac{x^8}{81}\right)\cdot\left(2x^2-\frac{3}{x^2}\right)^6$ is equal to : **[NA April 12, 2019 (II)]**

(a) –72 (b) 36
(c) –36 (d) –108

63. If ${}^{20}C_1+(2^2)\,{}^{20}C_2+(3^2)\,{}^{20}C_3+\ldots\ldots\ldots+(20^2)\,{}^{20}C_{20}=A(2^\beta)$, then the ordered pair (A, β) is equal to : **[April 12, 2019 (II)]**

(a) (420, 19) (b) (420, 18)
(c) (380, 18) (d) (380, 19)

64. The coefficient of x^{18} in the product $(1+x)(1-x)^{10}(1+x+x^2)^9$ is : **[April 12, 2019 (I)]**

(a) 84 (b) –126
(c) –84 (d) 126

65. If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1+ax+bx^2)(1-3x)^{15}$ in powers of x, then the ordered pair (a, b) is equal to: **[April 10, 2019 (I)]**

(a) (28, 861) (b) (–54, 315)
(c) (28, 315) (d) (–21, 714)

66. The sum of the series $2\cdot{}^{20}C_0+5\cdot{}^{20}C_1+8\cdot{}^{20}C_2+11\cdot{}^{20}C_3+\ldots+62\cdot{}^{20}C_{20}$ is equal to : **[April 8, 2019 (I)]**

(a) 2^{26} (b) 2^{25}
(c) 2^{23} (d) 2^{24}

67. The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is : **[Jan. 11, 2019 (I)]**

(a) 0 (b) 6
(c) 4 (d) 8

68. The value of r for which ${}^{20}C_r\,{}^{20}C_0+{}^{20}C_{r-1}\,{}^{20}C_1+{}^{20}C_{r-2}\,{}^{20}C_2+...+{}^{20}C_0\,{}^{20}C_r$ is maximum, is : **[Jan. 11, 2019 (I)]**

(a) 15 (b) 20
(c) 11 (d) 10

69. If $\sum_{r=0}^{25}\left\{{}^{50}C_r\cdot{}^{50-r}C_{25-r}\right\}=K\left({}^{50}C_{25}\right)$, then K is equal to: **[Jan. 10, 2019 (II)]**

(a) $(25)^2$ (b) $2^{25}-1$
(c) 2^{24} (d) 2^{25}

70. The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ **[Jan. 09, 2019 (II)]**

(a) 14 (b) 15
(c) 10 (d) 12

71. The value of $({}^{21}C_1-{}^{10}C_1)+({}^{21}C_2-{}^{10}C_2)+({}^{21}C_3-{}^{10}C_3)+({}^{21}C_4-{}^{10}C_4)+\ldots+({}^{21}C_{10}-{}^{10}C_{10})$ is : **[2017]**

(a) $2^{20}-2^{10}$ (b) $2^{21}-2^{11}$
(c) $2^{21}-2^{10}$ (d) $2^{20}-2^9$

72. If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is : **[2016]**

(a) 243 (b) 729
(c) 64 (d) 2187

73. The sum of coefficients of integral power of x in the binomial expansion $\left(1-2\sqrt{x}\right)^{50}$ is : **[2015]**

(a) $\frac{1}{2}\left(3^{50}-1\right)$ (b) $\frac{1}{2}\left(2^{50}+1\right)$
(c) $\frac{1}{2}\left(3^{50}+1\right)$ (d) $\frac{1}{2}\left(3^{50}\right)$

74. The coefficient of x^{1012} in the expansion of $(1+x^n+x^{253})^{10}$, (where $n \leq 22$ is any positive integer), is **[Online April 19, 2014]**

(a) 1 (b) ${}^{10}C_4$
(c) 4n (d) ${}^{253}C_4$

75. Let $S_1=\sum_{j=1}^{10} j(j-1)\,{}^{10}C_J$, $S_2=\sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3=\sum_{j=1}^{10} j^2\,{}^{10}C_j$. **[2010]**

Statement -1 : $S_3=55\times 2^9$.
Statement - 2: $S_1=90\times 2^8$ and $S_2=10\times 2^8$.

(a) Statement -1 is true, Statement -2 is true ; Statement -2 is **not** a correct explanation for Statement -1.
(b) Statement -1 is true, Statement -2 is false.
(c) Statement -1 is false, Statement -2 is true .
(d) Statement - 1 is true, Statement 2 is true ; Statement -2 is a correct explanation for Statement -1.

76. In a shop there are five types of ice-creams available. A child buys six ice-creams.

Statement-1 : The number of different ways the child can buy the six ice-creams is ${}^{10}C_5$.

Statement -2 : The number of different ways the child can buy the six ice-creams is equal to the number of different ways of arranging 6 A's and 4 B's in a row. **[2008]**

(a) Statement -1 is false, Statement-2 is true
(b) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1
(c) Statement -1 is true, Statement-2 is true; Statement -2 is not a correct explanation for Statement-1
(d) Statement -1 is true, Statement-2 is false

77. The sum of the series **[2007]**

${}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + ... - ..... + {}^{20}C_{10}$ is

(a) 0 (b) ${}^{20}C_{10}$
(c) $-{}^{20}C_{10}$ (d) $\frac{1}{2}\,{}^{20}C_{10}$

78. If x is so small that x^3 and higher powers of x may be neglected, then $\dfrac{(1+x)^{\frac{3}{2}}-\left(1+\frac{1}{2}x\right)^3}{(1-x)^{\frac{1}{2}}}$ may be approximated as **[2005]**

(a) $1-\frac{3}{8}x^2$ (b) $3x+\frac{3}{8}x^2$
(c) $-\frac{3}{8}x^2$ (d) $\frac{x}{2}-\frac{3}{8}x^2$

79. If x is positive, the first negative term in the expansion of $(1+x)^{27/5}$ is **[2003]**

(a) 6th term (b) 7th term
(c) 5th term (d) 8th term

80. The positive integer just greater than $(1+0.0001)^{10000}$ is **[2002]**

(a) 4 (b) 5
(c) 2 (d) 3

81. If the sum of the coefficients in the expansion of $(a+b)^n$ is 4096, then the greatest coefficient in the expansion is **[2002]**

(a) 1594 (b) 792
(c) 924 (d) 2924

Hints & Solutions

1. **(d)** $\frac{3^{200}}{8}=\frac{1}{8}(9^{100})$

$$=\frac{1}{8}(1+8)^{100}=\frac{1}{8}\left[1+n\cdot 8+\frac{n(n+1)}{2}\cdot 8^2+....\right]$$

$$=\frac{1}{8}+\text{Integer}$$

$$\therefore \left\{\frac{3^{200}}{8}\right\}=\left\{\frac{1}{8}+\text{integer}\right\}=\frac{1}{8}$$

2. **(13)**

$$T_{r+1}={}^{22}C_r\cdot(x^m)^{22-r}\cdot\left(\frac{1}{x^2}\right)^r$$

$$T_{r+1}={}^{22}C_r\cdot x^{22m-mr-2r}$$

$\because 22m-mr-2r=1$

$$\Rightarrow r=\frac{22m-1}{m+2}\Rightarrow r=22-\frac{3\cdot 3\cdot 5}{m+2}$$

So, possible value of $m=1, 3, 7, 13, 43$

But ${}^{22}C_r=1540$

$\therefore$ Only possible value of $m=13$.

3. **(120.00)**

Coefficient of x^4 in $\left(\frac{1-x^4}{1-x}\right)^6$ = coefficient of x^4 in $(1-6x^4)(1-x)^{-6}$

= coefficient of x^4 in $(1-6x^4)\left[1+{}^6C_1x+{}^7C_2x^2+....\right]$

$$={}^9C_4-6\cdot 1=126-6=120.$$

4. **(8.00)**

The given expression is $(2x^2+3x+4)^{10}=\sum_{r=0}^{20}a_rx^r$

General term $=\frac{10!}{r_1!r_2!r_3!}(2x^2)^{r_1}(3x)^{r_2}(4)^{r_3}$

Since, a_7 = Coeff. of x^7

$2r_1+r_2=7$ and $r_1+r_2+r_3=10$

Possibilities are

r_1	r_2	r_3
0	7	3
1	5	4
2	3	5
3	1	6

$$a_7=\frac{10!3^7 4^3}{7!3!}+\frac{10!(2)(3)^5(4)^4}{5!4!}+\frac{10!(2)^2(3)^3(4)^5}{2!3!5!}+\frac{10!(2)^3(3)(4)^6}{3!6!}$$

a_{13} = Coeff. of x^{13}

$2r_1+r_2=13$ and $r_1+r_2+r_3=10$

Possibilities are

r_1	r_2	r_3
3	7	0
4	5	1
5	3	2
6	1	3

$$a_{13}=\frac{10!(2^3)(3^7)}{3!7!}+\frac{10!(2^4)(3^5)(4)}{4!5!}+\frac{10!(2^5)(3^3)(4^2)}{5!\,3!\,2!}+\frac{10!(2^6)(3)(4^3)}{6!\,1!\,3!}$$

$$\therefore \frac{a_7}{a_{13}}=2^3=8$$

5. **(d)** Using Binomial expansion

$(x+a)^n+(x-a)^n=2(T_1+T_3+T_5+T_7...)$

$$\therefore\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6=2(T_1+T_3+T_5+T_7)$$

$2[{}^6C_0x^6+{}^6C_2x^4(x^2-1)+{}^6C_4x^2(x^2-1)^2+{}^6C_6(x^2-1)^3]$

$=2[x^6+15(x^6-x^4)+15x^2(x^4-2x^2+1)+(-1+3x^2-3x^4+x^6)]$

$=2(32x^6-48x^4+18x^2-1)$

$\alpha=-96$ and $\beta=36$

$\therefore\quad \alpha-\beta=-132$

6. (a) $\left(x^2+\frac{1}{x^2}\right)^n$

General term $T_{r+1} = {}^nC_r\left(x^2\right)^{n-r}\left(\frac{1}{x^3}\right)^r = {}^nC_r \cdot x^{2n-5r}$

To find coefficient of x, $2n-5r=1$

Given ${}^nC_r = {}^nC_{23} \Rightarrow r=23$ or $n-r=23$

$\therefore\ n=58$ or $n=38$

Minimum value is $n=38$

7. (b) $\because T_4 = 20\times 8^7$

$\Rightarrow {}^6C_3\left(\frac{2}{x}\right)^3 \times (x^{\log_8 x})^3 = 20\times 8^7$

$\Rightarrow 8\times 20\times\left(\frac{x^{\log_8 x}}{x}\right)^3 = 20\times 8^7 \Rightarrow \frac{x^{\log_8 x}}{x} = 64$

Now, take $\log_8$ on both sides, then

$(\log_8 x)^2 - (\log_8 x) = 2$

$\Rightarrow \log_8 x = -1$ or $\log_8 x = 2$

$\Rightarrow x = \frac{1}{8}$ or $x = 8^2$

8. (b) Given ${}^nC_{r-1} : {}^nC_r : {}^nC_{r+1} = 2:15:70$

$\Rightarrow \frac{{}^nC_{r-1}}{{}^nC_r} = \frac{2}{15}$ and $\frac{{}^nC_r}{{}^nC_{r+1}} = \frac{15}{70}$

$\Rightarrow \frac{r}{n-r+1} = \frac{2}{15}$ and $\frac{r+1}{n-r} = \frac{3}{14}$

$\Rightarrow 17r = 2n+2$ and $17r = 3n-14$

i.e., $2n+2 = 3n-14 \Rightarrow n=16$ & $r=2$

$\therefore$ Average $= \frac{{}^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3}$

$= \frac{696}{3} = 232$

9. (d) $(x+\sqrt{x^3-1})^6 + (x-\sqrt{x^3-1})^6$

$= 2[{}^6C_0x^6 + {}^6C_2x^4(x^3-1) + {}^6C_4x^2(x^3-1)^2 + {}^6C_6(x^3-1)^3]$

$= 2[x^6 + 15x^7 - 15x^4 + 15x^8 - 30x^5 + 15x^2 + x^9 - 3x^6 + 3x^3 - 1]$

Hence, the sum of coefficients of even powers of $x = 2[1-15+15+15-3-1] = 24$

10. (b) $\therefore$ fourth term is equal to 200.

$T_4 = {}^6C_3\left(\sqrt{x^{\left(\frac{1}{1+\log_{10}x}\right)}}\right)^3\left(x^{\frac{1}{12}}\right)^3 = 200$

$\Rightarrow 20x^{\frac{3}{2(1+\log_{10}x)}}.x^{\frac{1}{4}} = 200$

$x^{\frac{1}{4}+\frac{3}{2(1+\log_{10}x)}} = 10$

Taking $\log_{10}$ on both sides and putting $\log_{10}x = t$

$\left(\frac{1}{4}+\frac{3}{2(1+t)}\right)t = 1 \Rightarrow t^2+3t-4=0$

$\Rightarrow t^2+4t-t-4=0 \Rightarrow t(t+4)-1(t+4)=0$

$\Rightarrow t=1$ or $t=-4$

$\log_{10}x = 1 \Rightarrow x=10$

or $\log_{10}x = -4 \Rightarrow x = 10^{-4}$

According to the question $x>1$, $\therefore\ x=10$.

11. (c) $(x+10)^{50} + (x-10)^{50}$

$= a_0 + a_1x + a_2x^2 + \ldots + a_{50}x^{50}$

$\therefore\ a_0 + a_1x + a_2x^2 + \ldots + a_{50}x^{50}$

$= 2({}^{50}C_0x^{50} + {}^{50}C_2x^{48}\,.\,10^2 + {}^{50}C_4x^{46}\,.\,10^4 + \ldots)$

$\therefore\ a_0 = 2.{}^{50}C_{50}10^{50}$

$a_2 = 2.{}^{50}C_2.10^{48}$

$\therefore\ \frac{a_2}{a_0} = \frac{{}^{50}C_2\times 10^{48}}{{}^{50}C_{50}10^{50}}$

$= \frac{50\times 49}{2\times 100} = \frac{49}{4} = 12.25$

12. (a) Third term of $(1+x^{\log_2 x})^5 = {}^5C_2(x^{\log_2 x})^{5-3}$

$= {}^5C_2(x^{\log_2 x})^2$

Given, ${}^5C_2(x^{\log_2 x})^2 = 2560$

$\Rightarrow\ (x^{\log_2 x})^2 = 256 = (\pm 16)^2$

$\Rightarrow\ x^{\log_2 x} = 16$ or $x^{\log_2 x} = -16$ (rejected)

$\Rightarrow\ x^{\log_2 x} = 16 \Rightarrow \log_2 x \log_2 x = \log_2 16 = 4$

$\Rightarrow\ \log_2 x = \pm 2 \Rightarrow x = 2^2$ or 2^{-2}

$\Rightarrow\ x = 4$ or $\frac{1}{4}$

13. (a) Since, coefficient of x^2 in the expression $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)$ is a constant term, then

Coefficient of x^2 in $x^2\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

= co-efficient of constant term in $\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10}$

General term in $\left(\sqrt{x}+\frac{\lambda}{x^2}\right)^{10} = {}^{10}C_r\left(\sqrt{x}\right)^{10-r}\left(\frac{\lambda}{x^2}\right)^r$

$= {}^{10}C_r (x)^{\frac{10-r}{2}-2r} \cdot \lambda^2$

Then, for constant term,

$\frac{10-r}{2} - 2r = 0 \Rightarrow r = 2$

Co-efficient is x^2 in expression $= {}^{10}C_2\lambda^2 = 720$

$\Rightarrow \lambda^2 = \frac{720}{5\times 9} = 16 \Rightarrow \lambda = 4d$

Hence, required value of λ is 4.

14. **(b)** $2^{403} = 2^{400} \cdot 2^3$

$= 2^{4\times 100} \cdot 2^3$

$= (2^4)^{100} \cdot 8$

$= 8(2^4)^{100} = 8(16)^{100}$

$= 8\,(1+15)^{100}$

$= 8 + 15\,\mu$

When 2^{403} is divided by 15, then remainder is 8.

Hence, fractional part of the number is $\frac{8}{15}$

Therefore value of k is 8

15. **(a)** $\because (1+x)^2 = 1 + 2x + x^2$,

$(1+x^2)^3 = 1 + 3x^2 + 3x^4 + x^6$,

and $(1+x^3)^4 = 1 + 4x^3 + 6x^6 + 4x^9 + x^{12}$

So, the possible combinations for x^{10} are:

$x\,.\,x^9, x\,.\,x^6\,.\,x^3, x^2\,.\,x^2\,.\,x^6, x^4\,.\,x^6$

Corresponding coefficients are $2\times 4, 2\times 1\times 4, 1\times 3\times 6, 3\times 6$ or 8, 8, 18, 18.

$\therefore$ Sum of the coefficient is $8 + 8 + 18 + 18 = 52$

Therefore, the coefficient of x^{10} in the expansion of $(1+x)^2\,(1+x^2)^3\,(1+x^3)^4$ is 52.

16. **(d)** $\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8 + \left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8$

After rationalise the polynomial we get

$$\left[\frac{1}{\sqrt{5x^3+1}-\sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1}+\sqrt{5x^3-1}}{\sqrt{5x^3+1}+\sqrt{5x^3-1}}\right]^8 + \left[\frac{1}{\sqrt{5x^3+1}+\sqrt{5x^3-1}} \times \frac{\sqrt{5x^3+1}-\sqrt{5x^3-1}}{\sqrt{5x^3+1}-\sqrt{5x^3-1}}\right]^8$$

$$= \left[\frac{\sqrt{5x^3+1}+\sqrt{5x^3-1}}{(5x^3+1)-(5x^3-1)}\right]^8 + \left[\frac{\sqrt{5x^3+1}-\sqrt{5x^3-1}}{(5x^3+1)-(5x^3-1)}\right]^8$$

$$= \frac{1}{2^8}\left[\left(\sqrt{5x^3+1}+\sqrt{5x^3-1}\right)^8 + \left(\sqrt{5x^3+1}-\sqrt{5x^3-1}\right)^8\right]$$

$$= \frac{1}{2^8}\left[\begin{array}{l} {}^8C_0\left(\sqrt{5x^3+1}\right)^8 + {}^8C_2\left(\sqrt{5x^3+1}\right)^6\left(\sqrt{5x^3-1}\right)^2 \\ + {}^8C_4\left(\sqrt{5x^3+1}\right)^4\left(\sqrt{5x^3-1}\right)^4 \\ + {}^8C_6\left(\sqrt{5x^3+1}\right)^2\left(\sqrt{5x^3-1}\right)^6 + {}^8C_8\left(\sqrt{5x^3-1}\right)^8 \end{array}\right]$$

$$= \frac{1}{2^8}\left[\begin{array}{l} {}^8C_0(5x^3+1)^4 + {}^8C_2\,(5x^3+1)^3\,(5x^3-1) + 8_{C_4} \\ (5x^3+1)^2\,(5x^3-1)^2 \\ + {}^8C_6(5x^3+1)\,(5x^3-1)^3 + 8_{C_8}(5x^3-1)^4 \end{array}\right]$$

So, the degree of polynomial is 12,

Now, coefficient of $x^{12} = [{}^8C_0 5^4 + {}^8C_2 5^4 + {}^8C_4 5^4 + {}^8C_6 5^4$

$+ {}^8C_8 5^4]$

$= 5^4 \times \frac{2^8}{2} = 5^4 \times 2^4 \times \frac{2^4}{2}$

$= 10^4 \times 2^3 = 8\,(10)^4$

17. **(a)** Let $a = ((1+2x+3x^2)^6 + (1-4x^2)^6)$

$\therefore$ Coefficient of x^2 in the expansion of the product

$(2-x^2)\,((1+2x+3x^2)^6 + (1-4x^2)^6)$

$= 2$ (Coefficient of x^2 in a) $- 1$ (Constant of expansion)

In the expansion of $((1+2x+3x^2)^6 + (1-4x^2)^6)$.

Constant $= 1 + 1 = 2$

Coefficient of $x^2 =$ [Coefficient of x^2 in $({}^6C_0\,(1+2x)^6\,(3x^2)^0)$] + [Cofficient of x^2 in $({}^6C_1\,(1+2x)^5\,(3x^2)^1)$]

$-[{}^6C_1\,(4x^2)]$

$= 60 + 6\times 3 - 24 = 54$

$\therefore$ The coefficient of x^2 in $(2-x^2)\,((1+2x+3x^2)^6 +$

$(1-4x^2)^6)$

$= 2\times 54 - 1\,(2) = 108 - 2 = 106$

18. **(c)** Since we know that,

$(x+a)^5 + (x-a)^5$

$= 2[{}^5C_0 x^5 + {}^5C_2 x^3 \cdot a^2 + {}^5C_4 x \cdot a^4]$

$\therefore \left(x+\sqrt{x^3-1}\right)^5 + \left(x-\sqrt{x^3-1}\right)^5$

$= 2[{}^5C_0 x^5 + {}^5C_2 x^3(x^3-1) + {}^5C_4 x(x^3-1)^2]$

$\Rightarrow 2[x^5 + 10x^6 - 10x^3 + 5x^7 - 10x^4 + 5x]$

$\therefore$ Sum of coefficients of odd degree terms $= 2$.

19. (a) $\left[\frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{(x^{2/3}-x^{1/3}+1)}-\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$

$= \left(x^{1/3}+1-1-1/x^{1/2}\right)^{10} = \left(x^{1/3}-1/x^{1/2}\right)^{10}$

$T_{r+1} = {}^{10}C_r\ x^{\frac{20-5r}{6}}$
for r = 10

$T_{11} = {}^{10}C_{10}\ x^{-5}$

Coefficient of $x^{-5} = {}^{10}C_{10}(1)(-1)^{10} = 1$

20. (d) $\frac{(28-1)^{999}}{7} = \frac{28\lambda-1}{7}$

$\Rightarrow \frac{28\lambda-7+7-1}{7} = \frac{7(4\lambda-1)+6}{7}$

$\therefore$ Remainder = 6

21. (b) $T_{r+1} = {}^{18}C_r\left(x^{\frac{1}{3}}\right)^{18-r}\left(\frac{1}{2x^{\frac{1}{3}}}\right)^r = {}^{18}C_r x^{6-\frac{2r}{3}}\frac{1}{2^r}$

$\left\{\begin{array}{l} 6-\frac{2r}{3} = -2 \Rightarrow r = 12 \\ \& \ 6-\frac{2r}{3} = -4 \Rightarrow r = 15 \end{array}\right\}$

$\Rightarrow \frac{\text{coefficient of } x^{-2}}{\text{coefficient of } x^{-4}} = \frac{{}^{18}C_{12}\frac{1}{2^{12}}}{{}^{18}C_{15}\frac{1}{2^{15}}} = 182$

22. (c) $\frac{{}^nC_r}{1} = \frac{{}^nC_{r+1}}{7} = \frac{{}^nC_{r+2}}{42}$

By solving we get r = 6
so, it is 7th term.

23. (b) Consider $(1+ax+bx^2)(1-2x)^{18}$

$= (1+ax+bx^2)[{}^{18}C_0 - {}^{18}C_1(2x) + {}^{18}C_2(2x)^2 - {}^{18}C_3(2x)^3 + {}^{18}C_4(2x)^4 - ....]$

Coeff. of $x^3 = {}^{18}C_3(-2)^3 + a.(-2)^2.{}^{18}C_2 + b(-2).{}^{18}C_1 = 0$

Coeff. of $x^3 = -{}^{18}C_3.8 + a\times 4.\ {}^{18}C_2 - 2b\times 18 = 0$

$= -\frac{18\times17\times16}{6}.8 + \frac{4a+18\times17}{2} - 36b = 0$

$= -51\times16\times8 + a\times36\times17 - 36b = 0$

$= -34\times16 + 51a - 3b = 0$

$= 51a - 3b = 34\times16 = 544$

$= 51a - 3b = 544$ ....(i)

Only option number (b) satisfies the equation number (i)

24. (b) $4^n - 3n - 1 = (1+3)^n - 3n - 1$

$= [{}^nC_0 + {}^nC_1.3 + {}^nC_2.3^2 + ... + {}^nC_n3^n] - 3n - 1$

$= 9[{}^nC_2 + {}^nC_3.3 + ... + {}^nC_n.3^{n-2}]$

$\therefore 4^n - 3n - 1$ is a multiple of 9 for all n.

$\therefore X = \{x : x$ is a multiple of 9$\}$

Also, $Y = \{9(n-1) : n \in \mathbf{N}\}$

= {All multiples of 9}

Clearly $X \subset Y$. $\therefore X \cup Y = Y$

25. (a) $1 + x^4 + x^5 = \sum_{i=0}^{5} a_i(1+x)^i$

$= a_0 + a_1(1+x)^1 + a_2(1+x)^2 + a_3(1+x)^3 + a_4(1+x)^4 + a_5(1+x)^5$

$\Rightarrow 1 + x^4 + x^5$

$= a_0 + a_1(1+x) + a_2(1+2x+x^2) + a_3(1+3x+3x^2+x^3) + a_4(1+4x+6x^2+4x^3+x^4) + a_5(1+5x+10x^2+10x^3+5x^4+x^5)$

$\Rightarrow 1 + x^4 + x^5$

$= a_0 + a_1 + a_1x + a_2 + 2a_2x + a_2x^2 + a_3 + 3a_3x + 3a_3x^2 + a_3x^3 + a_4 + 4a_4x + 6a_4x^2 + 4a_4x^3 + a_4x^4 + a_5 + 5a_5x + 10a_5x^2 + 10a_5x^3 + 5a_5x^4 + a_5x^5$

$\Rightarrow 1 + x^4 + x^5$

$= (a_0 + a_1 + a_2 + a_3 + a_4 + a_5) + x(a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5) + x^2(a_2 + 3a_3 + 6a_4 + 10a_5) + x^3(a_3 + 4a_4 + 10a_5) + x^4(a_4 + 5a_5) + x^5(a_5)$

On comparing the like coefficients, we get

$\boxed{a_5 = 1}$ ...(i) ; $\boxed{a_4 + 5a_5 = 1}$ ...(ii);

$\boxed{a_3 + 4a_4 + 10a_5 = 0}$ ...(iii)

and $\boxed{a_2 + 3a_3 + 6a_4 + 10a_5 = 0}$ ...(iv)

from (i) & (ii), we get

$\boxed{a_4 = -4}$ ...(v) from (i), (iii) & (v), we get

$\boxed{a_3 = +6}$ ...(vi)

Now, from (i), (v) and (vi), we get

$a_2 = -4$

26. (a) Let r^{th} and $(r+1)^{th}$ term has equal coefficient

$\left(2+\frac{x}{3}\right)^{55} = 2^{55}\left(1+\frac{x}{6}\right)^{55}$

r^{th} term $= 2^{55}\ {}^{55}C_r\left(\frac{x}{6}\right)^r$

Coefficient of x^r is $2^{55}\ {}^{55}C_r \dfrac{1}{6^r}$

$(r+1)^{\text{th}}$ term $= 2^{55}\ {}^{55}C_{r+1}\left(\dfrac{x}{6}\right)^{r+1}$

Coefficient of x^{r+1} is $2^{55}\ {}^{55}C_{r+1}\cdot\dfrac{1}{6^{r+1}}$

Both coefficients are equal

$$2^{55}\ {}^{55}C_r \frac{1}{6^r} = 2^{55}\ {}^{55}C_{r+1}\frac{1}{6^{r+1}}$$

$$\frac{1}{\lfloor r \lfloor 55-r} = \frac{1}{\lfloor r+1 \lfloor 54-r}\cdot\frac{1}{6}$$

$6(r+1) = 55 - r$

$6r + 6 = 55 - r$

$7r = 49$

$r = 7$

$(r+1) = 8$

Coefficient of 7^{th} and 8^{th} terms are equal.

27. **(c)** Given expansion is

$(1+x)^{101}(1-x+x^2)^{100}$

$= (1+x)(1+x)^{100}(1-x+x^2)^{100}$

$= (1+x)[(1+x)(1-x+x^2)]^{100}$

$= (1+x)[(1-x^3)^{100}]$

Expansion $(1-x^3)^{100}$ will have $100 + 1 = 101$ terms

So, $(1+x)(1-x^3)^{100}$ will have $2 \times 101 = 202$ terms

28. **(a)** Expansion of $(1+x)^{2n}$ is $1 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \ldots\ldots + {}^{2n}C_r x^r + {}^{2n}C_{r+1} x^{r+1} + \ldots\ldots + {}^{2n}C_{2n} x^{2n}$

As given ${}^{2n}C_{r+2} = {}^{2n}C_{3r}$

$$\Rightarrow \frac{(2n)!}{(r+2)!(2n-r-2)!} = \frac{(2n)!}{(3r)!(2n-3r)!}$$

$\Rightarrow (3r)!(2n-3r)! = (r+2)!(2n-r-2)!$ ...(1)

Now, put value of n from the given choices.

Choice (a) put $n = 2r+1$ in (1)

LHS : $(3r)!(4r+2-3r)! = (3r)!(r+2)!$

RHS : $(r+2)!(3r)!$

$\Rightarrow$ LHS = RHS

29. **(d)** $(2^{1/2} + 3^{1/5})^{10} = {}^{10}C_0(2^{1/2})^{10} + {}^{10}C_1(2^{1/2})^9(3^{1/5}) + \ldots\ldots + {}^{10}C_{10}(3^{1/5})^{10}$

There are only two rational terms – first term and last term.

Now sum of two rational terms

$= (2)^5 + (3)^2 = 32 + 9 = 41$

30. **(b)** Let $r+1 = 7 \Rightarrow r = 6$

Given expansion is

$$\left(\frac{3}{\sqrt[3]{84}} + \sqrt{3}\ln x\right)^9, x > 0$$

We have

$T_{r+1} = {}^nC_r (x)^{n-r} a^r$ for $(x+a)^n$.

$\therefore$ According to the question

$$729 = {}^9C_6\left(\frac{3}{\sqrt[3]{84}}\right)^3 .(\sqrt{3}\ln x)^6$$

$$\Rightarrow 3^6 = 84 \times \frac{3^3}{84} \times 3^3 \times (6\ln x)$$

$\Rightarrow (\ln x)^6 = 1 \Rightarrow (\ln x)^6 = (\ln e)^6$

$\Rightarrow x = e$

31. **(a)** Consider $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$

$$= 2\left[{}^{2n}C_1\left(\sqrt{3}\right)^{2n-1} + {}^{2n}C_3\left(\sqrt{3}\right)^{2n-3} + {}^{2n}C_5\left(\sqrt{3}\right)^{2n-5} + \ldots\right]$$

$\because (a+b)^n - (a-b)^n$

$= 2[{}^nC_1 a^{n-1} b + {}^nC_3 a^{n-3} b^3 \ldots]$

= which is an irrational number.

32. **(a)** Given expansion is $\left(y^{\frac{1}{5}} + x^{\frac{1}{10}}\right)^{55}$

The general term is

$$T_{r+1} = {}^{55}C_r\left(y^{\frac{1}{5}}\right)^{55-r}.\left(x^{\frac{1}{10}}\right)^r$$

T_{r+1} would free from radical sign if powers of y and x are integers.

i.e. $\dfrac{55-r}{5}$ and $\dfrac{r}{10}$ are integer.

$\Rightarrow r$ is multiple of 10.

Hence, $r = 0, 10, 20, 30, 40, 50$

It is an A.P.

Thus, $50 = 0 + (k-1)\,10$

$50 = 10k - 10 \Rightarrow k = 6$

Thus, the six terms of the given expansion in which x and y are free from radical signs.

33. **(d)** Given function is

$f(y) = 1-(y-1)+(y-1)^2-(y-1)^3+........-(y-1)^{17}$

In the expansion of $(y-1)^n$

$T_{r+1} = {}^nC_r y^{n-r}(-1)^r$

coeff of y^2 in $(y-1)^2 = {}^2C_0$

coeff of y^2 in $(y-1)^3 = -{}^3C_1$

coeff of y^2 in $(y-1)^4 = {}^4C_2$

So, coeff of termwise is

${}^2C_0 + {}^3C_1 + {}^4C_2 + {}^5C_3 + ... + {}^{17}C_{15}$

$= 1 + {}^3C_1 + {}^4C_2 + {}^5C_3 + ... + {}^{17}C_{15}$

$= ({}^3C_0 + {}^3C_1) + {}^4C_2 + {}^5C_3 + ... + {}^{17}C_{15}$

$= {}^4C_1 + {}^4C_2 + {}^5C_3 + ... + {}^{17}C_{15}$

$= {}^5C_2 + {}^5C_3 + ... + {}^{17}C_{15}$

$= {}^{18}C_{15} = {}^{18}C_3$

34. **(a)** **Statement 2 :**

$P(n): n^7 - n$ is divisible by 7

Put $n = 1$, $1 - 1 = 0$ is divisible by 7, which is true

Let $n = k$, $P(k): k^7 - k$ is divisible by 7, true

Put $n = k + 1$

$\therefore$ $P(k+1): (k+1)^7 - (k+1)$ is div. by 7

$P(k+1): k^7 + {}^7C_1 k^6 + {}^7C_2 k^2 + ...+ {}^7C_6 k + 1 - k - 1$, is div. by 7.

$P(k+1): (k^7 - k) + ({}^7C_1 k^6 + {}^7C_2 k^5 + ...+ {}^7C_6 k)$ is div. by 7.

Since 7 is coprime with 1, 2, 3, 4, 5, 6.

So ${}^7C_1, {}^7C_2, ...{}^7C_6$ are all divisible by 7

$\therefore$ $P(k+1)$ is divisible by 7

Hence $P(n): n^7 - n$ is divisible by 7

Statement 1 : $n^7 - n$ is divisible by 7

$\Rightarrow$ $(n+1)^7 - (n+1)$ is divisible by 7

$\Rightarrow$ $(n+1)^7 - n^7 - 1 + (n^7 - n)$ is divisible by 7

$\Rightarrow$ $(n+1)^7 - n^7 - 1$ is divisible by 7

Hence both Statements 1 and 2 are correct and Statement 2 is the correct explanation of Statement -1.

35. **(b)** $(1 - x - x^2 + x^3)^6 = [(1-x) - x^2(1-x)]^6$

$= (1-x)^6 (1-x^2)^6$

$= (1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6) \times (1 - 6x^2 + 15x^4 - 20x^6 + 15x^8 - 6x^{10} + x^{12})$

Coefficient of $x^7 = (-6)(-20) + (-20)(15) + (-6)(-6)$

$= -144$

36. **(a)** $(8)^{2n} - (62)^{2n+1}$

$= (64)^n - (62)^{2n+1}$

$= (63+1)^n - (63-1)^{2n+1}$

$$= \left[{}^nC_0(63)^n + {}^nC_1(63)^{n-1} + {}^nC_2(63)^{n-2} + ...+ {}^nC_{n-1}(63) + {}^nC_n\right]$$

$$-\left[{}^{2n+1}C_0(63)^{2n+1} - {}^{2n+1}C_1(63)^{2n} + {}^{2n+1}C_2(63)^{2n-1} - ....+ (-1)^{2n+1}\,{}^{2n+1}C_{2n+1}\right]$$

$$= 63 \times \left[{}^nC_0(63)^{n-1} + {}^nC_1(63)^{n-2} + {}^nC_2(63)^{n-3} + ...+ {}^nC_{n-1}\right] + 1 - 63 \times \left[{}^{2n+1}C_0(63)^{2n} - {}^{2n+1}C_1(63)^{2n-1} + ...+ {}^{2n+1}C_{2n}\right] + 1$$

$= 63 \times$ some integral value $+ 2$

Hence, when divided by 9 leaves 2 as the remainder.

37. **(b)** **From statement 2:**

$$\sum_{r=0}^{n}(r+1)\,{}^nC_r x^r = \sum_{r=0}^{n} r.{}^nC_r x^r + \sum_{r=0}^{n}{}^nC_r x^r$$

$$= \sum_{r=1}^{n} r \cdot \frac{n}{r}\,{}^{n-1}C_{r-1}x^r + (1+x)^n$$

$$= nx\sum_{r=1}^{n}{}^{n-1}C_{r-1}x^{r-1} + (1+x)^n$$

$= nx(1+x)^{n-1} + (1+x)^n =$ RHS

$\therefore$ Statement 2 is correct.

Putting $x = 1$, we get

$$\sum_{r=0}^{n}(r+1)^n C_r = n \cdot 2^{n-1} + 2^n = (n+2)\cdot 2^{n-1}.$$

$\therefore$ Statement 1 is also true and statement 2 is a correct explanation for statement 1.

38. **(b)** $T_{r+1} = (-1)^r . {}^nC_r (a)^{n-r} . (b)^r$ is an expansion of $(a-b)^n$

$\therefore$ 5th term $= t_5 = t_{4+1}$

$= (-1)^4 . {}^nC_4 (a)^{n-4} . (b)^4 = {}^nC_4 . a^{n-4} . b^4$

6th term $= t_6 = t_{5+1} = (-1)^5\, {}^nC_5 (a)^{n-5} (b)^5$

Given $t_5 + t_6 = 0$

$\therefore$ ${}^nC_4 . a^{n-4} . b^4 + (-{}^nC_5 . a^{n-5} . b^5) = 0$

$$\Rightarrow \frac{n!}{4!(n-4)!} \cdot \frac{a^n}{a^4} . b^4 - \frac{n!}{5!(n-5)!} \cdot \frac{a^n b^5}{a^5} = 0$$

$$\Rightarrow \frac{n!.a^n b^4}{4!(n-5)!.a^4}\left[\frac{1}{(n-4)} - \frac{b}{5.a}\right] = 0 \quad [\because a \neq 0, b \neq 0]$$

$$\Rightarrow \frac{1}{n-4} - \frac{b}{5a} = 0 \Rightarrow \frac{a}{b} = \frac{n-4}{5}$$

39. **(d)** $(1-y)^m(1+y)^n$

$= [1 - {}^mC_1 y + {}^mC_2 y^2 - ......]\ [1 + {}^nC_1 y + {}^nC_2 y^2 + ...]$

$= 1 + (n-m)y + \left\{\frac{m(m-1)}{2} + \frac{n(n-1)}{2} - mn\right\}y^2 + ...$

$\therefore\ a_1 = n - m = 10$ ...(i)

and $a_2 = \frac{m^2 + n^2 - m - n - 2mn}{2} = 10$

$(m-n)^2 - (m+n) = 20$

$\Rightarrow\ m + n = 80$ ...(ii) [from (i)]

Solving (i) and (ii), we get

$\therefore\ m = 35, n = 45$

40. **(d)** T_{r+1} in the expansion

$\left[ax^2 + \frac{1}{bx}\right]^{11} = {}^{11}C_r (ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$

$= {}^{11}C_r\ (a)^{11-r}(b)^{-r}(x)^{22-2r-r}$

For the Coefficient of x^7, we have

$22 - 3r = 7 \Rightarrow r = 5$

$\therefore$ Coefficient of x^7

$= {}^{11}C_5 (a)^6\ (b)^{-5}$ ...(i)

Again T_{r+1} in the expansion

$\left[ax - \frac{1}{bx^2}\right]^{11} = {}^{11}C_r (ax^2)^{11-r}\left(-\frac{1}{bx^2}\right)^r$

$= {}^{11}C_r (a)^{11-r}(-1)^r \times (b)^{-r}(x)^{-2r+11-r}$

For the Coefficient of x^{-7}, we have

Now $11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$

$\therefore$ Coefficient of x^{-7}

$= {}^{11}C_6\, a^5 \times 1 \times (b)^{-6}$ ...(ii)

$\because$ Coefficient of x^7 = Coefficient of x^{-7}

From (i) and (ii),

$\therefore\ {}^{11}C_5\,(a)^6\,(b)^{-5} = {}^{11}C_6\, a^5 \times (b)^{-6}$

$\Rightarrow ab = 1.$

41. **(b)** Coeff. of x^n in $(1+x)(1-x)^n$

= coeff of x^n in

$(1+x)(1 - {}^nC_1 x + {}^nC_2 x^2 - .... + (-1)^n\,{}^nC_n x^n)$

$= (-1)^n\,{}^nC_n + (-1)^{n-1}\,{}^nC_{n-1}$

$= (-1)^n + (-1)^{n-1}.n$

$= (-1)^n(1-n)$

42. **(c)** $T_{r+1} = {}^{256}C_r (\sqrt{3})^{256-r}(\sqrt[8]{5})^r$

$= {}^{256}C_r (3)^{\frac{256-r}{2}}(5)^{r/8}$

Terms will be integral if $\frac{256-r}{2}$ & $\frac{r}{8}$ both are +ve integer.

It is possible if r is an integral multiple of 8 and $0 \le r \le 256$

$\therefore\ r = 33$

43. **(c)** $t_{r+2} = {}^{2n}C_{r+1}\ x^{r+1}; t_{3r} = {}^{2n}C_{3r-1}\ x^{3r-1}$

Given that, ${}^{2n}C_{r+1} = {}^{2n}C_{3r-1}$;

$\Rightarrow r + 1 + 3r - 1 = 2n$

$\Rightarrow 2n = 4r \Rightarrow n = 2r$

44. **(a)** We know that $t_{p+1} = {}^{p+q}C_p\, x^p$ and

$t_{q+1} = {}^{p+q}C_q\, x^q$

$\because\ {}^{p+q}C_p = {}^{p+q}C_q$. [Remember ${}^nC_r = {}^nC_{n-r}$]

45. **(c)** General term $= T_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r}\cdot\left(-\frac{k}{x^2}\right)^r$

$= {}^{10}C_r(-k)^r \cdot x^{\frac{10-r}{2} - 2r}$

$= {}^{10}C_r(-k)^r \cdot x^{\frac{10-5r}{2}}$

Since, it is constant term, then

$\frac{10-5r}{2} = 0 \Rightarrow r = 2$

$\therefore {}^{10}C_2(-k)^2 = 405$

$\Rightarrow k^2 = \frac{405 \times 2}{10 \times 9} = \frac{81}{9} = 9$

$\therefore |k| = 3$

46. **(a)** Consider the three consecutive coefficients of $(1+x)^{n+5}$ be ${}^{n+5}C_r, {}^{n+5}C_{r+1}, {}^{n+5}C_{r+2}$

$\because \frac{{}^{n+5}C_r}{{}^{n+5}C_{r+1}} = \frac{1}{2}$ (Given)

$\Rightarrow \frac{r+1}{n+5-r} = \frac{1}{2} \Rightarrow 3r = n + 3$ ...(i)

and $\frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_{r+2}} = \frac{5}{7}$

$\Rightarrow \frac{r+2}{n+4-r} = \frac{5}{7} \Rightarrow 12r = 5n + 6$ ...(ii)

Solving (i) and (ii) we get $r = 4$ and $n = 6$

$\therefore$ Largest coefficient in the expansion is ${}^{11}C_6 = 462.$

47. **(c)** Here, $\left(3^{\frac{1}{2}}+5^{\frac{1}{8}}\right)^n$

$$T_{r+1}={}^nC_r(3)^{\frac{n-r}{2}}(5)^{\frac{r}{8}}$$

$\because \frac{n-r}{2}$ and $\frac{r}{8}$ are integer

So, r must be 0, 8, 16, 24 ......

Now $n=t_{33}=a+(n-1)d=0+32\times 8=256$

$\Rightarrow n=256$

48. **(c)** General term $=T_{r+1}={}^9C_r\left(\frac{3x^2}{2}\right)^{9-r}\left(-\frac{1}{3x}\right)^r$

$$={}^9C_r\left(\frac{3}{2}\right)^{9-r}\left(-\frac{1}{3}\right)^r x^{18-3r}$$

The term is independent of x, then

$18-3r=0\Rightarrow r=6$

$$\therefore T_7={}^9C_6\left(\frac{3}{2}\right)^3\left(-\frac{1}{3}\right)^6={}^9C_3\left(\frac{1}{6}\right)^3$$

$$=\frac{9\times 8\times 7}{3\times 2\times 1}\left(\frac{1}{6}\right)^3=\left(\frac{7}{18}\right).$$

$$\therefore 18k=18\times\frac{7}{18}=7.$$

49. **(a)** General term of

$$(\alpha x^{\frac{1}{9}}+\beta x^{\frac{-1}{6}})^{10}={}^{10}C_r(\alpha x^{\frac{1}{9}})^{10-r}(\beta x^{\frac{-1}{6}})^r$$

$$={}^{10}C_r\alpha^{10-r}\beta^r(x)^{\frac{10-r}{9}-\frac{r}{6}}$$

Term independent of x if $\frac{10-r}{9}-\frac{r}{6}=0\Rightarrow r=4.$

$\therefore$ Term independent of $x={}^{10}C_4\alpha^6\beta^4$

Since $\alpha^3+\beta^2=4$

Then, by AM-GM inequality

$$\frac{\alpha^3+\beta^2}{2}\geq(\alpha^3b^2)^{\frac{1}{2}}$$

$\Rightarrow(2)^2\geq\alpha^3\beta^2\Rightarrow\alpha^6\beta^4\leq 16$

$\because$ The maximum value of the term independent of $x=10k$

$\therefore 10k={}^{10}C_4\cdot 16\Rightarrow k=336.$

50. **(118)**

According to the question,

${}^nC_{r-1}:{}^nC_r:{}^nC_{r+1}=2:5:12$

$\Rightarrow\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{5}{2}\Rightarrow\frac{n-r+1}{r}=\frac{5}{2}$

$\Rightarrow 2n-7r+2=0$ ...(i)

$\frac{{}^nC_{r+1}}{{}^nC_r}=\frac{12}{5}\Rightarrow\frac{n-r}{r+1}=\frac{12}{5}$

$\Rightarrow 5n-17r-12=0$ ...(ii)

Solving eqns. (i) and (ii),

$n=118, r=34$

51. **(b)** General term of the given expansion

$$T_{r+1}={}^{16}C_r\left(\frac{x}{\sin\theta}\right)^{16-r}\left(\frac{1}{x\cos\theta}\right)^r$$

For $r=8$ term is free from 'x'

$$T_9={}^{16}C_8\frac{1}{\sin^8\theta\cos^8\theta}$$

$$T_9={}^{16}C_8\frac{2^8}{(\sin 2\theta)^8}$$

When $\theta\in\left[\frac{\pi}{8},\frac{\pi}{4}\right]$, then least value of the term independent of x,

$l_1={}^{16}C_8 2^8$ [$\because$ min. value of l_1 at $\theta=\pi/4$]

When $\theta\in\left[\frac{\pi}{16},\frac{\pi}{8}\right]$, then least value of the term independent of x,

$$l_2={}^{16}C_8=\frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8}={}^{16}C_8.2^8.2^4$$

[$\because$ min. value of l_2 at $\theta=\pi/8$]

Now, $\frac{l_2}{l_1}=\frac{{}^{16}C_8.2^8.2^4}{{}^{16}C_8.2^8}=16$

52. **(d)** Let the general term of the expansion

$$T_{r+1}={}^{60}C_r\left(7^{\frac{1}{5}}\right)^{60-r}\left(-3^{\frac{1}{10}}\right)^r$$

$$={}^{60}C_r\cdot(7)^{12-\frac{r}{5}}(-1)^r\cdot(3)^{\frac{r}{10}}$$

Then, for getting rational terms, r should be multiple of L.C.M. of (5, 10)

Then, r can be 0, 10, 20, 30, 40, 50, 60.

Since, total number of terms = 61

Hence, total irrational terms = 61 – 7 = 54

53. **(c)** $\left(2^{\frac{1}{3}}+\frac{1}{2(3)^{\frac{1}{3}}}\right)^{10} = {}^{10}C_0\left(2^{\frac{1}{3}}\right)^0\left(\frac{1}{2(3)^{1/3}}\right)^{10}+$

$\cdots + {}^{10}C_{10}\left(2^{\frac{1}{3}}\right)^{10}\left(\frac{1}{2(3)^{1/3}}\right)^0$

5^{th} term from beginning $T_5 = {}^{10}C_4\left(2^{\frac{1}{3}}\right)^6\frac{1}{\left(2.3^{\frac{1}{3}}\right)^4}$

and 5^{th} term from end $T_{11-5+1} = {}^{10}C_6\left(2^{\frac{1}{3}}\right)^4\left(\frac{1}{2.3^{\frac{1}{3}}}\right)^6$

$\therefore\quad T_5 : T_7 = {}^{10}C_4\left(2^{\frac{1}{3}}\right)^6\left(\frac{1}{2.3^{\frac{1}{3}}}\right)^4 : {}^{10}C_6\left(2^{\frac{1}{3}}\right)^4\left(\frac{1}{2.3^{\frac{1}{3}}}\right)^6$

$=\left(2^{\frac{1}{3}}\right)^2 : \left(\frac{1}{2.3^{\frac{1}{3}}}\right)^2$

$=\frac{2^{\frac{2}{3}}\cdot 2^2\cdot 3^{\frac{2}{3}}}{1} = 4(6)^{\frac{2}{3}} : 1 = 4.(36)^{\frac{1}{3}} : 1$

54. **(c)** General term of $\left(2x^2-\frac{1}{x}\right)^8$ is

${}^8C_r(2x^2)^{8-r}\left(\frac{-1}{x}\right)^r$

$\therefore$ Given expression is equal to

$\left(1-\frac{1}{x}+3x^5\right){}^8C_r(2x^2)^{8-r}\left(-\frac{1}{x}\right)^r$

$= {}^8C_r(2x^2)^{8-r}\left(-\frac{1}{x}\right)^r - \frac{1}{x}{}^8C_r(2x^2)^{8-r}\left(-\frac{1}{x}\right)^r$

$+3x^5.{}^8C_r(2x^2)^{8-r}\left(-\frac{1}{x}\right)^r$

$= {}^8C_r 2^{8-r}(-1)^r x^{16-3r} - {}^8C_r 2^{8-r}(-1)^r x^{15-3r}$

$+3.{}^8C_r 2^{(8-r)}\left(-\frac{1}{x}\right)^r(-1)^r x^{21-3r}$

For the term independent of x, we should have

$16-3r=0,\ 15-3r=0,\ 21-3r=0$

From the simplification we get r = 5 and r = 7

$\therefore -{}^8C_5(2^3)(-1)^5 - 3.{}^8C_7.2$

$+\left[\frac{8!}{5!3!}\times 8\right]-3\times\left[\frac{8!}{7!1!}\times 2\right]$

$=(56\times 8)-48$

$=448-6\times 8=448-48=400$

55. **(c)** Given expression can be written as

$\left[\frac{\left(x^{1/3}\right)^3+1^3}{x^{2/3}-x^{1/3}+1}-\frac{\left(\sqrt{x}\right)^2-1^2}{\sqrt{x}(\sqrt{x}-1)}\right]^{10}$

$=\left((x^{1/3}+1)-\left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10} = \left(x^{1/3}+1-1-\frac{1}{\sqrt{x}}\right)^{10}$

$=(x^{1/3}-x^{-1/2})^{10}$

General term $= T_{r+1}$

$= {}^{10}C_r(x^{1/3})^{10-r}(-x^{-1/2})^r = {}^{10}C_r\, x^{\frac{10-r}{3}}\cdot(-1)^r\cdot x^{-\frac{r}{2}}$

$= {}^{10}C_r(-1)^r\cdot x^{\frac{10-r}{3}-\frac{r}{2}}$

Term will be independent of x when $\frac{10-r}{3}-\frac{r}{2}=0$

$\Rightarrow\ r=4$

So, required term $= T_5 = {}^{10}C_4 = 210$

56. **(d)** $T_{r+1} = {}^{15}C_r(x^2)^{15-r}.(2x^{-1})^r = {}^{15}C_r\times(2)^r\times x^{30-3r}$

For independent term, $30-3r=0 \Rightarrow r=10$

Hence the term independent of x,

$T_{11} = {}^{15}C_{10}\times(2)^{10}$

For term involve x^{15}, $30-3r=15 \Rightarrow r=5$

Hence coefficient of $x^{15} = {}^{15}C_5\times(2)^5$

Required ratio $=\frac{{}^{15}C_5\times(2)^5}{{}^{15}C_{10}\times(2)^{10}} = \frac{\frac{15!}{10!5!}}{\frac{15!}{5!10!}\times(2)^5}$

$=1:32$

57. **(d)** Given expansion can be written as

$\left(\frac{x-1}{x}\right)^n.(1-x)^n = (-1)^n x^{-n}(1-x)^{2n}$

Total number of terms will be $2n + 1$ which is odd ($\because$ $2n$ is always even)

$\therefore$ Middle term $=\frac{2n+1+1}{2} = (n+1)$ th

Now, $T_{r+1} = {}^nC_r(1)^r x^{n-r}$

So, $\frac{{}^{2n}C_n\cdot x^{2n-n}}{x^n.(-1)^n} = {}^{2n}C_n.(-1)^n$

Middle term is an odd term. So, $n + 1$ will be odd.

So, n will be even.

$\therefore$ Required answer is ${}^{2n}C_n$.

58. **(c)** The middle term in the expansion of

$(1+\alpha x)^4 = T_3 = {}^4C_2(\alpha x)^2 = 6\alpha^2x^2$

The middle term in the expansion of

$(1-\alpha x)^6 = T_4 = {}^6C_3(-\alpha x)^3 = -20\alpha^3x^3$

According to the question

$6\alpha^2 = -20\alpha^3 \Rightarrow \alpha = -\frac{3}{10}$

59. **(a)** The given series, $\sum_{r=0}^{20} {}^{50-r}C_6$

$= {}^{50}C_6 + {}^{49}C_6 + {}^{48}C_6 + {}^{47}C_6 + ... + {}^{32}C_6 + {}^{31}C_6 + {}^{30}C_6$

$= ({}^{30}C_7 + {}^{30}C_6) + {}^{31}C_6 + {}^{32}C_6 + ... + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

$= ({}^{31}C_7 + {}^{31}C_6) + {}^{32}C_6 + ... + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

$= ({}^{32}C_7 + {}^{32}C_6) + ... + {}^{49}C_6 + {}^{50}C_6 - {}^{30}C_7$

....

....

....

$= {}^{51}C_7 - {}^{30}C_7$

60. **(615)** General term of the expansion $= \frac{10!}{\alpha!\beta!\gamma!}x^{\beta+2\gamma}$

For coefficient of x^4; $\beta + 2\gamma = 4$

Here, three cases arise

Case-1 : When $\gamma = 0, \beta = 4, \alpha = 6$

$\Rightarrow \frac{10!}{\alpha!\beta!\gamma!}x^{\beta+2\gamma}$

Case-2 : When $\gamma = 1, \beta = 2, \alpha = 7$

$\Rightarrow \frac{10!}{7!2!1!} = 360$

Case-3 : When $\gamma = 2, \beta = 0, \alpha = 8$

$\Rightarrow \frac{10!}{8!0!2!} = 45$

Hence, total = 615

61. **(30)** Let $(1-x+x^2 ..... x^{2n})(1+x+x^2 ..... x^{2n})$

$= a_0 + a_1x + a_2x^2 + ...$

put $x = 1$

$1(2n+1) = a_0 + a_1 + a_2 + ... a_{2n}$ ...(i)

put $x = -1$

$(2n+1) \times 1 = a_0 - a_1 + a_2 + ... a_{2n}$ ...(ii)

Adding (i) and (ii), we get,

$4n + 2 = 2(a_0 + a_2 + ...) = 2 \times 61$

$\Rightarrow 2n + 1 = 61 \Rightarrow n = 30.$

62. **(d)** Given expression is,

$\left(\frac{1}{60} - \frac{x^8}{81}\right)\left(2x^2 - \frac{3}{x^2}\right)^6$

$= \frac{1}{60}\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{x^8}{81}\left(2x^2 - \frac{3}{x^2}\right)^6$

Term independent of x,

= Coefficient of x° in $\frac{1}{60}\left(2x^2 - \frac{3}{x^2}\right)^6 - \frac{1}{81}$.

coefficient of x^{-8} in $\left(2x^2 - \frac{3}{x^2}\right)^6$

$= \frac{-1}{60}\, {}^6C_3 (2)^3 (3)^3 + \frac{1}{81}\, {}^6C_5 (2)(3)^5$

$= -72 + 36 = -36$

63. **(b)** Given, ${}^{20}C_1 + 2^2.{}^{20}C_2 + 3^2.{}^{20}C_3 + ... + 20^2.{}^{20}C_{20} = A(2^\beta)$

Taking L.H.S.,

$= \sum_{r=1}^{20} r^2.\, {}^{20}C_r = 20\sum_{r=1}^{20} r.\, {}^{19}C_{r-1}$

$= 20\left[\sum_{r=1}^{20}(r-1)\, {}^{19}C_{r-1} + \sum_{r=1}^{20} {}^{19}C_{r-1}\right]$

$= 20\left[19\sum_{r=2}^{20} {}^{18}C_{r-2} + 2^{19}\right] = 20[19 \,.\, 2^{18} + 2^{19}]$

$= 420 \times 2^{18}$

Now, compare it with R.H.S., A = 420 and $\beta = 18$

64. **(a)** Given expression,

$(1-x)^{10}(1+x+x^2)^9(1+x) = (1-x^3)^9(1-x^2)$

$= (1-x^3)^9 - x^2(1-x^3)^9$

$\Rightarrow$ Coefficient of x^{18} in $(1-x^3)^9$ – coeff. of x^{16} in $(1-x^3)^9$

$= {}^9C_6 - 0 = \frac{9!}{6!3!} = \frac{7\times8\times9}{6} = 84$

65. **(c)** Given expression is $(1+ax+bx^2)(1-3x)^{15}$

Co-efficient of $x^2 = 0$

$\Rightarrow {}^{15}C_2(-3)^2 + a\,.\,{}^{15}C_1(-3) + b\,.\,{}^{15}C_0 = 0$

$\Rightarrow \frac{15\times14}{2}\times9 - 15\times3a + b = 0$

$\Rightarrow 945 - 45a + b = 0$ ...(i)

Now, co-efficient of $x^3 = 0$

$\Rightarrow {}^{15}C_3(-3)^3 + a\,.\,{}^{15}C_2(-3)^2 + b\,.{}^{15}C_1(-3) = 0$

$\Rightarrow \frac{15\times14\times13}{3\times2}\times(-3\times3\times3) + a\times\frac{15\times14\times9}{2} - b\times3\times15 = 0$

$\Rightarrow 15\times3[-3\times7\times13 + a\times7\times3 - b] = 0$

$\Rightarrow 21a - b = 273$ ...(ii)

From (i) and (ii), we get,

$a = 28, b = 315 \Rightarrow (a, b) \equiv (28, 31, 5)$

66. **(b)** $2.^{20}C_0 + 5.^{20}C_1 + 8.^{20}C_2 + ... + 62.^{20}C_{20}$

$$= \sum_{r=0}^{20} (3r+2)\ ^{20}C_r = 3\sum_{r=0}^{20} r.\ ^{20}C_r + 2\sum_{r=0}^{20} {^{20}C_r}$$

$$= 60\sum_{r=1}^{20} {^{19}C_{n-1}} + 2\sum_{r=0}^{20} {^{20}C_r}$$

$= 60 \times 2^{19} + 2 \times 2^{20} = 2^{21}\ [15 + 1] = 2^{25}$

67. **(a)** Middle Term, $\left(\frac{n}{2}+1\right)^{th}$ term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ is,

$$T_{4+1} = {^8C_4}\left(\frac{x^3}{3}\right)^4\left(\frac{3}{x}\right)^4 = 5670$$

$\Rightarrow \frac{8\times7\times6\times5}{4\times3\times2} \times x^{12-4} = 5670$

$\Rightarrow x^8 = 81$

$\Rightarrow x^8 - 81 = 0$

$\therefore$ sum of all values of x = sum of roots of equation $(x^8 - 81 = 0)$.

68. **(b)** Consider the expression

$^{20}C_r\ ^{20}C_0 + {^{20}C_{r-1}}\ ^{20}C_1 + {^{20}C_{r-2}}\ ^{20}C_2 + ... + {^{20}C_0} \cdot {^{20}C_r}$

For maximum value of above expression r should be equal to 20.

as $^{20}C_{20} \cdot {^{20}C_0} + {^{20}C_{19}} \cdot {^{20}C_1} + ... + {^{20}C_{20}} \cdot {^{20}C_0}$

$= (^{20}C_0)^2 + (^{20}C_1)^2 + ... + (^{20}C_{20})^2 = {^{40}C_{20}}$.

Which is the maximum value of the expression,

So, $r = 20$.

69. **(d)** $$\sum_{r=0}^{25}\left({^{50}C_r}.{^{50-r}C_{25-r}}\right) = \sum_{r=0}^{25}\left(\frac{\lfloor 50}{\lfloor 50-r \lfloor r}\frac{\lfloor 50-r}{\lfloor 25 \lfloor 25-r}\right)$$

$$= \sum_{r=0}^{25}\left(\frac{\lfloor 50}{\lfloor 25}\times\frac{1}{\lfloor 25}\times\left(\frac{\lfloor 25}{\lfloor 25-r \lfloor r}\right)\right)$$

$$= {^{50}C_{25}}\sum_{r=0}^{25} {^{25}C_r} = {^{50}C_{25}}(2^{25})$$

Then, by comparison, $K = 2^{25}$

70. **(b)** Consider the expression

$$\left(\frac{1-t^6}{1-t}\right)^3 = (1-t^6)^3(1-t)^{-3}$$

$$= (1 - 3t^6 + 3t^{12} - t^{18})\left(1 + 3t + \frac{3\cdot4}{2!}t^2 + \frac{3\cdot4\cdot5}{3!}t^3 + \frac{3\cdot4\cdot5\cdot6}{4!}t^4 + ...\infty\right)$$

Hence, the coefficient of $t^4 = 1\cdot\frac{3\cdot4\cdot5\cdot6}{4!}$

$= \frac{3\times4\times5\times6}{4\times3\times2\times1} = 15$

71. **(a)** We have $(^{21}C_1 + {^{21}C_2}\ ...... + {^{21}C_{10}})$

$- (^{10}C_1 + {^{10}C_2}\ .....\ {^{10}C_{10}})$

$= \frac{1}{2}[(^{21}C_1 + ... + {^{21}C_{10}}) + (^{21}C_{11} + ...\ {^{21}C_{20}})] - (2^{10} - 1)$

$(\because\ {^{10}C_1} + {^{10}C_2} + ... + {^{10}C_{10}} = 2^{10} - 1)$

$= \frac{1}{2}[2^{21} - 2] - (2^{10} - 1)$

$= (2^{20} - 1) - (2^{10} - 1) = 2^{20} - 2^{10}$

72. **(b)** Total number of terms $= {^{n+2}C_2} = 28$

$(n + 2)(n + 1) = 56;\ n = 6$

$\therefore$ Put $x = 1$ in expansion $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^6$,

we get sum of coefficient $= (1 - 2 + 4)^6$

$= 3^6 = 729$.

73. **(c)** We know that $(a + b)^n + (a - b)^n$

$= 2[^nC_0a^nb^0 + {^nC_2}a^{n-2}b^2 + {^nC_4}a^{n-4}b^4 ...]$

$(1-2\sqrt{x})^{50} + (1+2\sqrt{x})^{50}$

$2\left[{^{50}C_0} + {^{50}C_2}(2\sqrt{x})^2 + {^{50}C_4}(2\sqrt{x})^4 ...\right]$

$= 2\left[{^{50}C_0} + {^{50}C_2}2^2x + {^{50}C_4}2^4x^2 + ...\right]$

Putting x = 1, we get,

$^{50}C_0 + {^{50}C_2}2^2 + {^{50}C_4}2^4 ... = \frac{3^{50}+1}{2}$

74. **(b)** Given expansion $(1 + x^n + x^{253})^{10}$

Let $x^{1012} = (1)^a\ (x^n)^b.\ (x^{253})^c$

Here a, b, c, n are all +ve integers and $a \le 10$, $b \le 10$, $c \le 4$, $n \le 22$, $a + b + c = 10$

Now $bn + 253c = 1012$

$\Rightarrow bn = 253\ (4 - c)$

For $c < 4$ and $n \le 22$; $b > 10$, which is not possible.

$\therefore c = 4, b = 0, a = 6$

$\therefore x^{1012} = (1)^6.\ (x^n)^0\ .\ (x^{253})^4$

Hence the coefficient of $x^{1012} = \frac{10!}{6!0!4!} = {^{10}C_4}$

75. **(b)** $S_2=\sum_{j=1}^{10} j\,{}^{10}C_j=\sum_{j=1}^{10}10\,{}^{9}C_{j-1}$

$\left[\because {}^{n}C_r=\frac{n}{r}{}^{n-1}C_{r-1}\right]$

$=10\left[{}^{9}C_0+{}^{9}C_1+{}^{9}C_2+....+{}^{9}C_9\right]=10.2^9$

76. **(a)** The given situation in statement 1 is equivalent to find the non negative integral solutions of the equation $x_1+x_2+x_3+x_4+x_5=6$
which is coeff. of x^6 in the expansion of
$(1+x+x^2+x^3+.....\infty)^5=$ coeff. of x^6 in $(1-x)^{-5}$

$=$ coeff. of x^6 in $1+5x+\frac{5.6}{2!}x^2$..........

$=\frac{5\cdot6\cdot7\cdot8\cdot9\cdot10}{6!}=\frac{10!}{6!4!}={}^{10}C_6$

$\therefore$ Statement 1 is wrong.
Number of ways of arranging 6A's and 4B's in a row

$=\frac{10!}{6!4!}={}^{10}C_6$ which is same as the number of ways the child can buy six icecreams.
$\therefore$ Statement 2 is true.

77. **(d)** We know that, $(1+x)^{20}={}^{20}C_0+{}^{20}C_1x+{}^{20}C_2x^2+......{}^{20}C_{10}x^{10}+.....{}^{20}C_{20}x^{20}$

Put $x=-1$, $(0)={}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+......+{}^{20}C_{10}-{}^{20}C_{11}....+{}^{20}C_{20}$

$\Rightarrow 0=2[{}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+.....-{}^{20}C_9]+{}^{20}C_{10}$

$\Rightarrow {}^{20}C_{10}=2[{}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+......-{}^{20}C_9+{}^{20}C_{10}]$

$\Rightarrow {}^{20}C_0-{}^{20}C_1+{}^{20}C_2-{}^{20}C_3+....+{}^{20}C_{10}$

$=\frac{1}{2}\,{}^{20}C_{10}$

78. **(c)** $\because$ x^3 and higher powers of x may be neglected

$$\therefore \frac{(1+x)^{\frac{3}{2}}-\left(1+\frac{x}{2}\right)^3}{\left(1-x^2\right)^{\frac{1}{2}}}$$

$$=(1-x)^{\frac{-1}{2}}\left[\left(1+\frac{3}{2}x+\frac{\frac{3}{2}.\frac{1}{2}}{2!}x^2\right)-\left(1+\frac{3x}{2}+\frac{3.2}{2!}\frac{x^2}{4}\right)\right]$$

$$=\left[1+\frac{x}{2}+\frac{\frac{1}{2}.\frac{3}{2}}{2!}x^2\right]\left[\frac{-3}{8}x^2\right]=\frac{-3}{8}x^2$$

79. **(d)** $T_{r+1}=\frac{n(n-1)(n-2).........(n-r+1)}{r!}(x)^r$

For first negative term,
$n-r+1<0\Rightarrow r>n+1$

$\Rightarrow r>\frac{32}{5}\ \therefore r=7.\left(\because n=\frac{27}{5}\right)$

Therefore, first negative term is T_8.

80. **(d)** $(1+0.0001)^{10000}=\left(1+\frac{1}{n}\right)^n, n=10000$

$=1+n.\frac{1}{n}+\frac{n(n-1)}{2!}\frac{1}{n^2}+\frac{n(n-1)(n-2)}{3!}\frac{1}{n^3}+...+\frac{1}{n^n}$

$=1+1+\frac{1}{2!}\left(1-\frac{1}{n}\right)+\frac{1}{3!}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)+...+\frac{1}{n^n}$

$<1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+........+\frac{1}{(9999)!}$

$=1+\frac{1}{1!}+\frac{1}{2!}+.......\infty=e<3$

81. **(c)** Take $a=1$ and $b=1$ in $(a+b)^n$.
$2^n=4096=2^{12}\Rightarrow n=12;$
The greatest coeff = coeff of middle term.
So middle term = t_7.
$\Rightarrow t_7=t_{6+1}={}^{12}C_6a^6b^6$

$\Rightarrow$ Coeff of $t_7={}^{12}C_6=\frac{12!}{6!6!}=924.$